

# Asymmetric Fermion Superfluid with Inter- and Intra-Species Pairings

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We investigate the phase structure of an asymmetric fermion superfluid with inter- and intra-species pairings. The introduction of the intra-species pairing mechanism in canonical ensemble changes significantly the phase diagram and brings in a new state with coexisting inter- and intra-species pairings. Different from the case with only inter-species pairing, all the fermion excitations are fully gapped in the region with intra-species pairing.

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Recently, the fermion pairing between different species with mismatched Fermi surfaces prompted great interest in both experimental[1, 2, 3] and theoretical[4, 5, 6, 7, 8, 9, 10, 11, 12, 13] studies. In conventional fermion superfluid the ground state is well described by the BCS theory, while for asymmetric fermion superfluid the phase structure is much more rich and the pairing mechanism is not yet very clear. Various exotic phases have been suggested, such as the Sarma phase[14] or gapless phase[15] where the superfluid component is breached by a normal component in momentum space[16], the Fulde-Ferrel-Larkin-Ovchinnikov(FFLO) phase[17] where the Cooper pair is momentum dependent, the phase with deformed Fermi surfaces where the two Fermi surfaces cross to each other and the total momentum zero Cooper pairs could form at the crossing node[18, 19], and the phase separation[20, 21] in coordinate space where the normal and superfluid components are inhomogeneously mixed.

For many fermion superfluid systems, the fermions from the same species can form Cooper pairs as well. In ultracold atom gas like  $^6\text{Li}$  and  $^{40}\text{K}$ , there exist pairings between different elements and between different states of the same element. In color superconducting phase of dense quark matter[22], the quarks of different flavors can form total spin zero Cooper pairs, and the quarks of the same flavor can be combined into total spin one pairs which describe better the cooling rates of quark stars[23]. In neutron stars[24], proton-proton, neutron-neutron and neutron-proton pairings are all possible. Recently discovered high temperature superconductivity of  $\text{MgB}_2$ [25, 26, 27, 28] can also be well described by an extended two-band BCS theory where the electrons from the same energy band form Cooper pairs.

Since the ratio of the intra- to inter-species pairing gap is already 10% for color superconductivity and can even reach 50% for nuclear superfluidity and ultracold atom gas, the competition between the two types of pairings should be strong. Considering that the pairing between fermions from the same species happens at the common Fermi surface, the intra-species pairing will become more favored than the inter-species pairing, when the Fermi surface mismatch between the two kinds of species is large enough. Therefore, some familiar phenomena in the superfluid with only inter-species pairing may be washed out by the introduction of intra-species pairing, and some new phases may arise. For instance, the interesting gapless state may disappear, the inhomogeneous FFLO state at low temperature[12, 29, 30] may be eaten up by the homogeneous intra-species pairing state, and the mixed state of BCS superfluid and normal phase[20, 21] may be replaced by a new phase where the inter- and intra-species pairings coexist. In this paper we propose a general model to investigate the competition between the inter- and intra-species pairings.

We consider a fermion fluid containing two species  $a$  and  $b$  with masses  $m_a$  and  $m_b$  and chemical potentials  $\mu_a$  and  $\mu_b$ . The system can be described by the Lagrangian density

$$\mathcal{L} = \sum_{i,\sigma} \bar{\psi}_i^\sigma \left[ -\frac{\partial}{\partial \tau} + \frac{\nabla^2}{2m_i} + \mu_i \right] \psi_i^\sigma - \frac{g}{2} \sum_{\sigma \neq \sigma', \rho \neq \rho'} \bar{\psi}_a^\sigma \bar{\psi}_b^{\sigma'} \psi_b^{\rho'} \psi_a^\rho - \sum_i g_i \bar{\psi}_i^\uparrow \bar{\psi}_i^\downarrow \psi_i^\downarrow \psi_i^\uparrow, \quad (1)$$

where  $\psi_i^\sigma(x)$  are fermion fields with  $i = a, b$  and (pseudo-)spin  $\sigma = \uparrow, \downarrow$ , the coupling constants  $g, g_a$  and  $g_b$  controlling respectively the inter- and intra-species pairings, are negative to keep the interactions attractive. The Lagrangian has the symmetry  $U_a(1) \otimes U_b(1)$  with the element  $U(\theta_a, \theta_b)$  defined as  $U(\theta_a, \theta_b) \psi_a^\sigma = e^{i\theta_a} \psi_a^\sigma$  and  $U(\theta_a, \theta_b) \psi_b^\sigma = e^{i\theta_b} \psi_b^\sigma$ .

We introduce the condensates of  $a$ - $a$ ,  $b$ - $b$  and  $a$ - $b$  pairs,  $\Phi_a = -g_a \langle \psi_a^\downarrow \psi_a^\uparrow \rangle$ ,  $\Phi_b = -g_b \langle \psi_b^\downarrow \psi_b^\uparrow \rangle$  and  $\Phi = -(g/2) \sum_{\sigma \neq \sigma'} \langle \psi_b^{\sigma'} \psi_a^\sigma \rangle$ . To incorporate the FFLO state in the study, we assume that the condensates are in the form of  $\Phi_a(\mathbf{x}) = \Delta_a e^{2i\mathbf{q}_a \cdot \mathbf{x}}$ ,  $\Phi_b(\mathbf{x}) = \Delta_b e^{2i\mathbf{q}_b \cdot \mathbf{x}}$  and  $\Phi(\mathbf{x}) = \Delta e^{i(\mathbf{q}_a + \mathbf{q}_b) \cdot \mathbf{x}}$  with  $\Delta_a, \Delta_b$  and  $\Delta$  being independent of  $\mathbf{x}$ . Obviously, the translational symmetry and rotational symmetry in the FFLO state are spontaneously broken. Note that, for the sake of simplicity, the FFLO state we considered here is its simplest pattern, namely the single plane wave FFLO state or the so-called FF state.

The key quantity of a thermodynamic system is the partition function  $\mathcal{Z}$  which can be represented by the path integral

$$\mathcal{Z} = \prod \int \mathcal{D}\psi_i^\sigma \mathcal{D}\bar{\psi}_i^\sigma \exp \left( \int d\tau d\mathbf{x} \mathcal{L} \right). \quad (2)$$

By performing a gauge transformation for the fermion fields,  $\chi_a^\sigma = e^{-i\mathbf{q}_a \cdot \mathbf{x}} \psi_a^\sigma$  and  $\chi_b^\sigma = e^{-i\mathbf{q}_b \cdot \mathbf{x}} \psi_b^\sigma$ , the path integral over  $\chi_{a,b}^\sigma$  in the partition function  $\mathcal{Z}$  of the system at mean field level can be calculated easily, and we obtain the mean field thermodynamic potential

$$\begin{aligned}\Omega &= -\frac{T}{V} \ln \mathcal{Z} \\ &= -\frac{\Delta_a^2}{g_a} - \frac{\Delta_b^2}{g_b} - \frac{2\Delta^2}{g} - T \sum_{n, \mathbf{p}} \text{Tr} \ln G^{-1}(i\omega_n, \mathbf{p})\end{aligned}\quad (3)$$

in the imaginary time formulism of finite temperature field theory, where  $\omega_n = (2n+1)\pi T$  is the fermion frequency,  $\mathbf{p}$  is the fermion momentum, and the inverse of the Nambu-Gorkov propagator can be written as

$$G^{-1} = \begin{pmatrix} i\omega_n - \epsilon_a^+ & 0 & \Delta & \Delta_a \\ 0 & i\omega_n - \epsilon_b^+ & \Delta_b & -\Delta \\ \Delta^* & \Delta_b^* & i\omega_n + \epsilon_b^- & 0 \\ \Delta_a^* & -\Delta^* & 0 & i\omega_n + \epsilon_a^- \end{pmatrix} \quad (4)$$

with  $\epsilon_i^\pm = (\mathbf{p} \pm \mathbf{q}_i)^2/(2m_i) - \mu_i$ .

The Lagrangian (1) is non-renormalizable, a regularization scheme should be applied. We use the s-wave scattering length regularization which relates the bare coupling constants  $g, g_a$  and  $g_b$  to the low energy limit of the corresponding two-body T-matrices in vacuum by

$$\begin{aligned}\frac{m}{4\pi a_a} &= \frac{1}{g_a} + \sum_{\mathbf{p}} \frac{m}{\mathbf{p}^2}, \\ \frac{m}{4\pi a_b} &= \frac{1}{g_b} + \sum_{\mathbf{p}} \frac{m}{\mathbf{p}^2}, \\ \frac{m}{4\pi a} &= \frac{1}{g} + \sum_{\mathbf{p}} \frac{m}{\mathbf{p}^2}\end{aligned}\quad (5)$$

with the s-wave scattering lengths  $a_a, a_b$  and  $a$ . Such a scheme is reliable in the whole region of interacting strength, and therefore it is possible to extend our study to the BCS-BEC crossover, although we focus in this paper only on the weak coupling BCS region. For simplicity, we have taken the same particle masses  $m_a = m_b = m$  and will take all the condensates as real numbers.

One should note that if the relative momentum  $\mathbf{q}_a - \mathbf{q}_b$  is large enough, the uniform superfluid would be unstable due to the stratification of the superfluid components characterized by  $\Phi_a(\mathbf{x})$  and  $\Phi_b(\mathbf{x})$ . The case here is analogous to the multi-component BEC. To avoid such a dynamic instability[31, 32, 33], we choose  $\mathbf{q}_a = \mathbf{q}_b = \mathbf{q}$ [34]. After computing the frequency summation and taking a Bogoliubov-Valatin transformation from particles  $a$  and  $b$  to quasi-particles, the thermodynamic potential can be expressed in terms of the quasi-particles,

$$\Omega = -\frac{m\Delta_a^2}{4\pi a_a} - \frac{m\Delta_b^2}{4\pi a_b} - \frac{2m\Delta^2}{4\pi a} + (\Delta_a^2 + \Delta_b^2 + 2\Delta^2) \sum_{\mathbf{p}} \frac{m}{\mathbf{p}^2} + \sum_{\mathbf{p}} \sum_{i=a,b} \epsilon_i^- - \sum_{\mathbf{p}} \sum_{j,k=\pm} \left[ \frac{E_j^k}{2} + T \ln \left( 1 + e^{-E_j^k/T} \right) \right], \quad (6)$$

where  $E_\pm^\mp$  are the quasi-particle energies

$$E_\pm^\mp = \sqrt{\epsilon_\pm^2 + \delta\epsilon^2} \pm \sqrt{\epsilon_\pm^4 + \epsilon_\pm^4} \mp \delta\epsilon \quad (7)$$

with  $\epsilon_\pm, \delta\epsilon$  and  $\epsilon_\Delta$  defined as

$$\begin{aligned}\epsilon_\pm^2 &= [(\epsilon_a^+ \epsilon_a^- + \Delta_a^2 + \Delta^2) \pm (\epsilon_b^+ \epsilon_b^- + \Delta_b^2 + \Delta^2)]/2, \\ \delta\epsilon &= (\epsilon_a^+ - \epsilon_a^-)/2 = (\epsilon_b^+ - \epsilon_b^-)/2, \\ \epsilon_\Delta^4 &= \Delta^2[(\epsilon_a^+ - \epsilon_b^+)(\epsilon_a^- - \epsilon_b^-) + (\Delta_b - \Delta_a)^2].\end{aligned}\quad (8)$$

$\Delta_a \neq 0$  and  $\Delta_b \neq 0$  correspond, respectively, to the spontaneous symmetry breaking patterns  $U_a(1) \otimes U_b(1) \rightarrow U_b(1)$  and  $U_a(1)$ , and  $\Delta \neq 0$  means the breaking pattern  $U_a(1) \otimes U_b(1) \rightarrow U_{a-b}(1)$  with the element  $U(\theta)$  defined as  $U(\theta)\psi_a^\sigma = e^{i\theta}\psi_a^\sigma$  and  $U(\theta)\psi_b^\sigma = e^{-i\theta}\psi_b^\sigma$ .

The condensates and the FFLO momentum as functions of temperature and chemical potentials are determined by the gap equations,

$$\frac{\partial \Omega}{\partial \Delta_a} = 0, \quad \frac{\partial \Omega}{\partial \Delta_b} = 0, \quad \frac{\partial \Omega}{\partial \Delta} = 0, \quad \frac{\partial \Omega}{\partial \mathbf{q}} = 0, \quad (9)$$

and the ground state of the system is specified by the minimum of the thermodynamic potential, namely by the second-order derivatives.

To see clearly the effect of the mismatch between the two species  $a$  and  $b$ , we introduce the average chemical potential  $\mu = (\mu_a + \mu_b)/2$  and the chemical potential mismatch  $\delta\mu = (\mu_b - \mu_a)/2$  instead of  $\mu_a$  and  $\mu_b$ . Without loss of generality, we assume  $\mu_b > \mu_a$ . We choose  $p_F a, p_F a_a, p_F a_b$  as the free parameters of the model, where  $p_F = \sqrt{2m\mu}$  is the average Fermi momentum, and set  $p_F a = -0.58$  and  $p_F a_a = p_F a_b = 0.73 p_F a$  in the numerical calculation. We have checked that in the BCS region of  $0 < p_F |a|, p_F |a_a|, p_F |a_b| < 1$  and  $0 < |a_a| = |a_b| < |a|$  to guarantee weak interaction for  $a$ - $b$  pairing and more weak coupling for  $a$ - $a$  and  $b$ - $b$  pairings, there is no qualitative change in the obtained phase diagrams. Note that, in the symmetric case with  $\delta\mu = 0$ , from a direct integration of the gap equation, one obtains the famous result[35]  $\Delta_0 \simeq 8\mu e^{-2} \exp[-\pi/(2p_F |a|)]$  for the inter-species pairing gap  $\Delta_0$  at zero temperature.

We first consider systems of grand canonical ensemble with fixed chemical potentials. The phase diagram in  $T - \delta\mu$  plane is shown in Fig.1. The left panel is the familiar case without intra-species pairing[36]. Both thermal fluctuations and large chemical potential mismatch can break the Cooper pairs. The homogeneous BCS state can exist at low temperature and low mismatch, and the inhomogeneous FFLO state survives only in a narrow mismatch window. The phase transition from the superfluid to normal phase is of second order, and the transition from the homogeneous to inhomogeneous superfluid at zero temperature is of first order and happens at  $\delta\mu_c = \Delta_0/\sqrt{2}$  [36]. When the intra-species pairing is included as well, see the right panel of Fig.1, the inhomogeneous FFLO state of  $a$ - $b$  pairing is eaten up by the homogeneous superfluid of  $a$ - $a$  and  $b$ - $b$  pairings at low temperature, just as we expected, and survives only in a small triangle at high temperature. The phase transition from the  $a$ - $b$  pairing superfluid to the  $a$ - $a$  and  $b$ - $b$  pairing superfluid is of first order. From the assumption of  $\mu_b > \mu_a$ , the temperature to melt the condensate  $\Delta_b$  is higher than that to melt  $\Delta_a$ , which leads to a phase with only  $b$ - $b$  pairing. Since  $\mu_b$  increases and  $\mu_a$  decreases with mismatch  $\delta\mu$ , the region with only  $b$ - $b$  pairing becomes more and more wide when  $\delta\mu$  increases. Note that, for systems with fixed chemical potentials there is no mixed phase of inter- and intra-species pairings, and the situation is similar to a three-component fermion system[37].

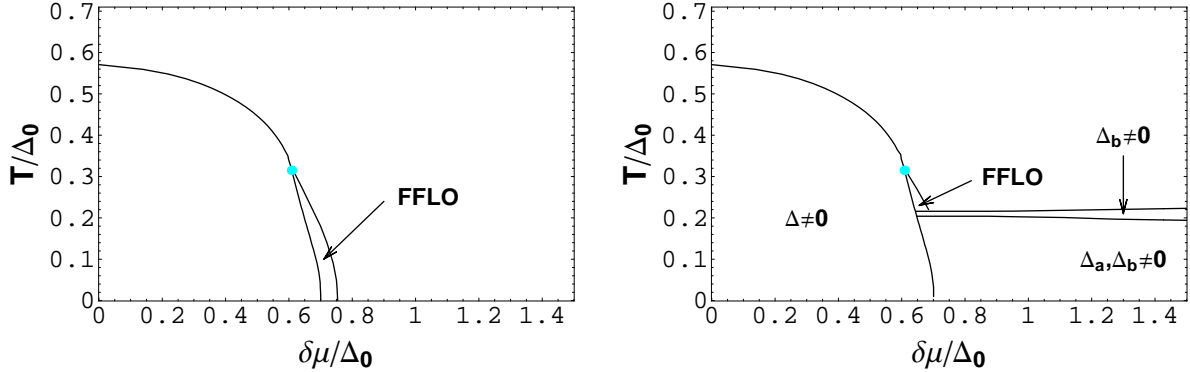


FIG. 1: The phase diagram of an asymmetric fermion superfluid in grand canonical ensemble.  $\delta\mu$  is the chemical potential mismatch between the two species, and the average chemical potential is fixed to be  $\mu = 50\Delta_0$  with  $\Delta_0$  being the corresponding symmetric gap at  $\delta\mu = 0$ . The left panel is for the familiar case with only inter-species pairing, and in the right panel the intra-species pairing is included as well.

For many physical systems, the fixed quantities are not chemical potentials  $\mu_a$  and  $\mu_b$  but particle number densities  $n_a = -\partial\Omega/\partial\mu_a$  and  $n_b = -\partial\Omega/\partial\mu_b$ , or equivalently speaking the total number density  $n = n_b + n_a$  and the number density asymmetry  $\alpha = \delta n/n$  with  $\delta n = n_b - n_a$ . Such systems are normally described by the canonical ensemble and the essential quantity is the free energy which is related to the thermodynamic potential by a Legendre transformation,  $\mathcal{F} = \Omega + \mu_a n_a + \mu_b n_b = \Omega + \mu n + \delta\mu \delta n$ .

It is easy to prove that the gap equations in the canonical ensemble are equivalent to (9) in the grand canonical ensemble and give the same solutions. While the candidates of the ground state of the system are the same in both ensembles, the stability conditions for the two ensembles are very different and lead to much more rich phase structure in systems with fixed number densities. Our method to obtain the phase diagram at fixed total number density  $n$  is as the following. We first calculate the gap equations (9) and obtain all the possible homogeneous phases and inhomogeneous FFLO phase, then compare their free energies to extract the lowest one at fixed  $T$  and  $\alpha$ , and finally investigate the stability of the system against number fluctuations by computing the number susceptibility matrix

$$\chi = \frac{\partial^2 \mathcal{F}}{\partial n_i \partial n_j} = -\frac{\partial^2 \Omega}{\partial \mu_i \partial \mu_j} + Y_i R^{-1} Y_j^\dagger, \quad (10)$$

where  $Y_i$  are susceptibility vectors with elements  $(Y_i)_m = \partial^2 \Omega / (\partial \mu_i \partial x_m)$  and  $R$  is a susceptibility matrix with elements  $R_{mn} = \partial^2 \Omega / (\partial x_m \partial x_n)$  in the order parameter space constructed by  $x = (\Delta_a, \Delta_b, \Delta, \mathbf{q})$ . The state with non positive-definite  $\chi$  (denoted by  $\chi < 0$ ) is unstable against number fluctuations and may be a phase separation[38].

The phase diagram without intra-species pairing can be found in [12, 29, 30]. As a comparison we recalculate it and show it in the left panel of Fig.2. By computing the gap equations (9) and then finding the minimum of the free energy  $\mathcal{F}$ , we find that the superfluid is in homogeneous state at high temperature and FFLO state at low temperature. However, the number susceptibility in the region of low temperature and low number asymmetry is not positive-definite, the FFLO state in this region is therefore unstable against the number fluctuations, and the ground state is probably an inhomogeneous mixture of the BCS superfluid and normal fermion fluid. The shadowed region is the gapless superfluid with  $\delta\mu > \Delta$  where the energy gap to excite quasi-particles is zero and the system may be sensitive to the thermal and quantum fluctuations. The phase diagram with both inter- and intra-species pairings is shown in the right panel of Fig.2. Besides the familiar phase with only inter-species pairing ( $\Delta \neq 0, \Delta_a = \Delta_b = 0$ ) and the expected phases with only intra-species pairing ( $\Delta_a, \Delta_b \neq 0, \Delta = 0$  and  $\Delta_b \neq 0, \Delta = \Delta_a = 0$ ), there appears a new phase where the two kinds of pairings coexist ( $\Delta, \Delta_a, \Delta_b \neq 0$ ). In this new phase the FFLO momentum is zero and the number susceptibility is negative,  $\chi < 0$ . Therefore, the homogeneous superfluid in this region is unstable against number fluctuations, and the ground state is probably a inhomogeneous mixture of these three superfluid components. In the familiar phase with only inter-species pairing, there remain a stable FFLO region and an unstable FFLO triangle where the number susceptibility is negative and the system may be in the state of phase separation. As we expected in the introduction, the gapless state appears only in the inter-species pairing superfluid, and in the region with intra-species pairing all the fermions are fully gapped.

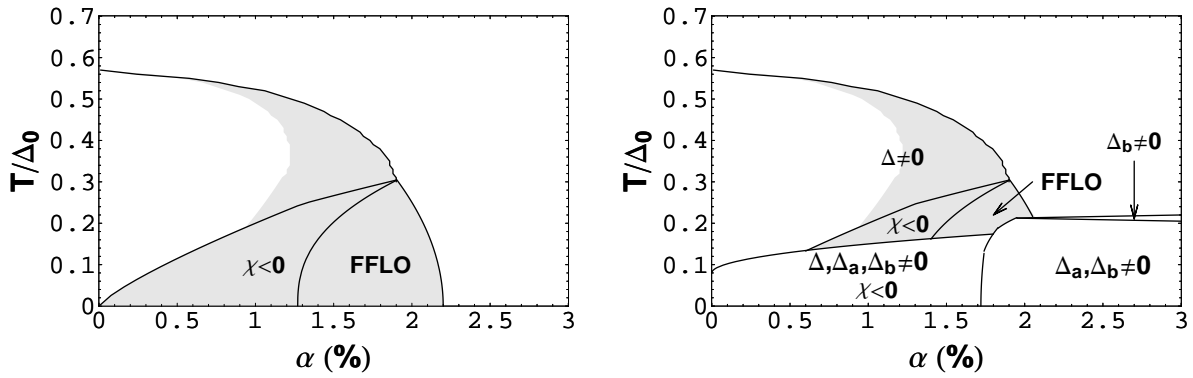


FIG. 2: The phase diagram of a mismatched fermion superfluid in canonical ensemble.  $\alpha$  describes the number asymmetry between the two species, and the total number  $n$  is fixed corresponding to  $\mu = 50\Delta_0$ . The left panel is for the familiar case with only inter-species pairing. In the right panel the intra-species pairing is included as well. The shadowed regions indicate the gapless superfluid.

In summary, we have investigated the phase structure of an asymmetric two-species fermion superfluid with both inter- and intra-species pairings. Since the attractive interaction for the intra-species pairing is relatively weaker, its introduction changes significantly the conventional phase diagram with only inter-species pairing at low temperature. For systems with fixed chemical potentials, the inhomogeneous superfluid with inter-species pairing at low temperature is replaced by the homogeneous superfluid with intra-species pairing, while for systems with fixed species numbers, the two kinds of pairings can coexist at low temperature and low number asymmetry. In any region with intra-species pairing, the interesting gapless superfluid is washed out and all fermion excitations are fully gapped. To finally determine the exact state of the system in the regions with negative number susceptibility ( $\chi < 0$ ) in Fig.2, further investigation, especially considering other possible inhomogeneous superfluid phases, is needed.

Since the intra-species pairing is significant at low temperature, our result including both inter- and intra-species pairings is expected to influence the characteristics of those low temperature fermion systems. For instance, it will change the equation of state of the nuclear superfluid (where neutron-proton and neutron-neutron, proton-proton pairings play the roles of inter- and intra-species pairings) or the color superconductor of quark matter (where up-down and up-up, down-down quark pairings play the roles of inter- and intra-species pairings in two flavor case) in compact stars. On the other hand, systems involving two different alkali atoms can be used to study the result we obtained.

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  - [38] Note that  $\chi_{ij} = dn_i/d\mu_j$  is always positive for uniform systems and  $\chi < 0$  is a signature of a phase separation.